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Effect of gear tooth faults on time varying mesh stiffness of spur gear pair

Ankur Saxena [a], Anand Parey [b]*, and Manoj Chouksey [c]

[a,b] *Mechanical Engineering Department, School of Engineering, Indian Institute of Technology, Indore, 453441, India*
[c] *Mechanical Engineering Department, Shri Govindram Seksaria Institute of Technology and Science, Indore, 452003, India*
* *Corresponding author. Tel.: +91 7324240716; fax: +91 7324240700; email: anandp@iiti.ac.in*

ABSTRACT

This paper presents, a computer simulation based analytical approach to quantify the time varying mesh stiffness (TVMS) reduction of gear pair due to various gear tooth faults. Gear faults affecting transmission of gear pair are always accompanied by a stiffness reduction. TVMS is an important parameter in condition monitoring and to understand the dynamics of meshing gear pair. Potential energy method is one of the most widely used analytical methods to calculate TVMS. In this paper, reduction of gear mesh stiffness has been studied for three gear faults: cracked tooth, chipped tooth and spalled tooth using potential energy method. The results show that due to presence of gear faults the TVMS reduces which will affect the vibration response of spur gear pair.

Keywords: Gear tooth faults, mesh stiffness, stiffness reduction.

1. Introduction

Vibration response of gear pair is closely related to TVMS of the gear pair. Two methods are being widely used by researchers to calculate TVMS of gear pair, viz. finite element method (FEM) and analytical method (AM). Tooth surface failure occurs due to excessive stress conditions. This can cause removal and/or plastic deformation of the contacting tooth surfaces leading to various gear faults like tooth root crack, tooth breakage, chipping, spalling, etc. Study of these gear faults on vibration responses is very important in condition monitoring of gear pair as it will help in identifying and reducing the vibration of overall system.

Many researchers have proposed their analytical methods to calculate the TVMS of healthy and damaged gears. Yang and Sun (1985) proposed a value of Hertzian energy which is further extended by Yang and Lin (1987) to calculate TVMS of a gear pair by the potential energy method by including bending energy and axial compressive energy with Hertzian energy. This model was further refined by Tian (2004) by taking the shear energy into consideration. Tian also discussed the effect of chipped tooth, cracked tooth and a broken tooth. Wu (2008) presented the refined model of Tian for faulty gear pair for calculating the total effective mesh stiffness as a function of crack length, crack intersection angle and rotation angle of gear for a pair of meshing spur gears consisting of a perfect gear and a pinion with cracked tooth. Chaari et al (2008) studied effect of spalling and tooth breakage on gear mesh stiffness and computed dynamic response using dynamic modeling. Chen and Shao (2011) proposed an analytical mesh stiffness model of spur gear with tooth root crack propagating along both tooth width and crack depth. Pandya and Parey (2013) have adopted principle of LEFM to carry out crack propagation path studies with different contact ratio and predicted the change in TVMS for different crack propagation path.

Investigation of gear mesh stiffness has been carried out extensively for healthy and cracked gears. But very less work is done on other gear tooth errors of spur gear pair. In this paper, the total effective mesh stiffness of mating teeth of the gear is investigated. Potential energy method is used to study the mesh stiffness reduction corresponding to three main localized gear faults such as tooth crack, tooth chip and tooth spall. For the pair of mating gear made up of same material it is observed that faults are usually observed at pinion tooth. Thus, in this paper only the faults present on the pinion tooth will be considered.

2. Mesh stiffness formulation

2.1 Mesh stiffness calculation of healthy gear tooth pair

The gear stiffness model used in this study is based on potential energy method used by Tian [3] in 2004. The energy stored in meshing gear system was assumed to include four components out of which three components: Hertzian energy, bending energy and axial compressive energy is given in Ref [2] and fourth component: shear energy was proposed in Ref [3]. Thus for the single – tooth pair meshing duration, the total effective TVMS can be expressed as [3].

$$k_t = \frac{1}{1/k_h + 1/k_{b1} + 1/k_{s1} + 1/k_{a1} + 1/k_{b2} + 1/k_{s2} + 1/k_{a2}} \quad (1)$$

where, k_h , k_b , k_s and k_a represents the Hertzian, bending, shear and axial compressive mesh stiffness respectively and subscripts 1 and 2 denote the driving and driven gears respectively and their values can be obtained in Ref. [3].

$$k_h = \frac{\pi EL}{4(1-\nu^2)} \tag{2}$$

$$\frac{1}{k_b} = \int_{-\alpha_1}^{\alpha_2} \frac{3\left[1+\cos\alpha_1\{(\alpha_2-\alpha)\sin\alpha-\cos\alpha\}\right]^2(\alpha_2-\alpha)\cos\alpha}{2EL[\sin\alpha+(\alpha_2-\alpha)\cos\alpha]^3}d\alpha \tag{3}$$

$$\frac{1}{k_a} = \int_{-\alpha_1}^{\alpha_2} \frac{(\alpha_2-\alpha)\cos\alpha\sin^2\alpha_1}{2EL[\sin\alpha+(\alpha_2-\alpha)\cos\alpha]}d\alpha \tag{4}$$

$$\frac{1}{k_s} = \int_{-\alpha_1}^{\alpha_2} \frac{1.2(1+\nu)(\alpha_2-\alpha)\cos\alpha\cos^2\alpha_1}{EL[\sin\alpha+(\alpha_2-\alpha)\cos\alpha]}d\alpha \tag{5}$$

Values of various gear tooth parameters used in calculating stiffness in equations (2)-(5) are represented in Figure 1.

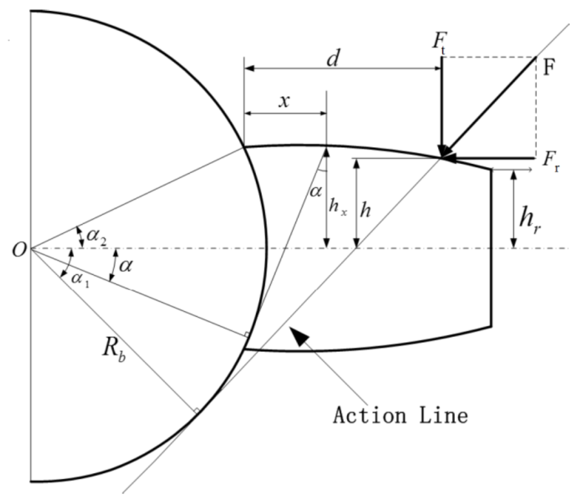

Figure 1 Schematic diagram of spur gear tooth model [3]

For the double-tooth-pair meshing duration, the total effective TVMS is the sum of the two pair's stiffness, which can be expressed as

$$k_t = \sum_{i=1}^{2} \frac{1}{1/k_{h,i} + 1/k_{b1,i} + 1/k_{s1,i} + 1/k_{a1,i} + 1/k_{b2,i} + 1/k_{s2,i} + 1/k_{a2,i}} \tag{6}$$

Where, i = 1 represents the first pair of meshing teeth and i = 2 represents the second pair of meshing teeth.

2.2 Mesh stiffness calculation of gear with a cracked tooth

If crack development has been initiated at the root of a single tooth of a pinion then the formula mentioned in equations (3), (5) is not valid because bending and shear stiffness will change due to influence of the crack. This phenomenon will occur because when the crack is present, the effective area moment of inertia and the area of the cross section will change. So for single tooth mesh period the total effective mesh stiffness is given by

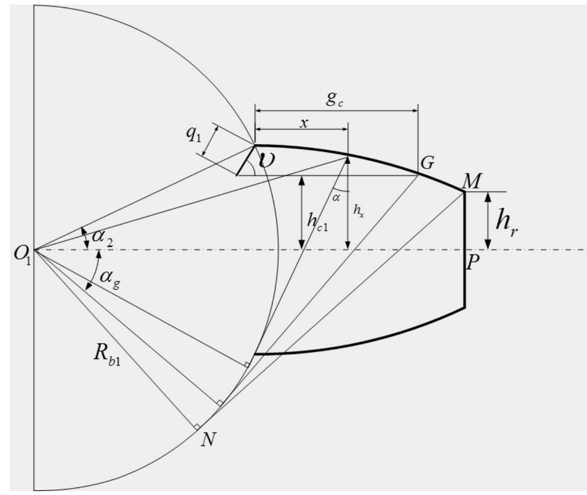

Figure 2 Schematic diagram of cracked tooth [3]

$$k_{t\_crack} = \frac{1}{1/k_h + 1/k_{b\_crack} + 1/k_{s\_crack} + 1/k_{a1} + 1/k_{b2} + 1/k_{s2} + 1/k_{a2}} \tag{7}$$

where, k_{b_crack} and k_{s_crack} are the bending and shear mesh stiffness of cracked tooth respectively and their values can be obtained in Ref. [3].

$$\frac{1}{k_{b\_crack}} = \int_{-\alpha_1}^{\alpha_2} \frac{12\{1+\cos\alpha_1[(\alpha_2-\alpha)\sin\alpha-\cos\alpha]\}^2(\alpha_2-\alpha)\cos\alpha}{EL[\sin\alpha_2-\frac{q_1}{R_{b1}}\sin\upsilon+\sin\alpha+(\alpha_2-\alpha)\cos\alpha]^3}d\alpha \tag{8}$$

$$\frac{1}{k_{s\_crack}} = \int_{-\alpha_1}^{\alpha_2} \frac{2.4(1+\nu)(\alpha_2-\alpha)\cos\alpha(cos\alpha_1)^2}{EL[\sin\alpha_2-\frac{q_1}{R_{b1}}\sin\upsilon+\sin\alpha+(\alpha_2-\alpha)\cos\alpha]}d\alpha \tag{9}$$

These values are used in equation (6) to calculate the stiffness of cracked gear tooth. Figure 2 shows the crack length 'q_1' and crack intersection angle 'v' which are decisive parameters and are responsible for stiffness reduction.

2.3 Mesh stiffness calculation of gear with a chipped tooth

Chipped tooth is a common fault usually observed when there are some unwanted particles present in between gear tooth mesh. In this study we have assumed that chip is so thin as compared with thickness of the gear tooth, that effect of bending, shear and axial compressive stiffness can be neglected. However the contact width of gear surface will change as the gear tooth rotates along the tooth curve causing change in Hertzian stiffness which is calculated as

$$K_{h_{chip}} = \frac{\pi EL_c}{4(1-v^2)} \tag{10}$$

Where, L_c = width of effective work surface and can be expressed as [3]

$$L_c = \begin{cases} L, & if\ 0 \le d_c \le d_h - b \\ \left(\frac{d_h^2 z}{b} - d_h z\right)\frac{1}{d_c} + L - \frac{d_h z}{b} & if\ d_h - b < d_c \le d_h \end{cases} \tag{11}$$

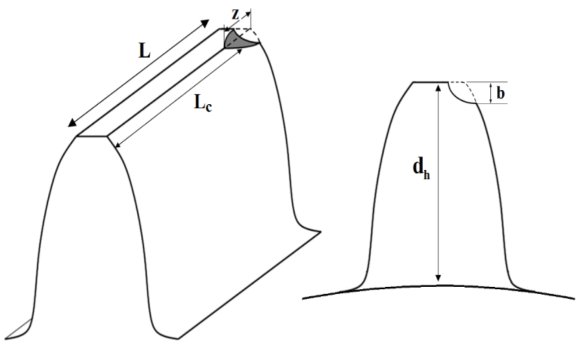

Figure 3. Schematic diagram of chipped tooth

Chipped profile is assumed as a segment of a hyperbolic curve. The values of d_h, z, b, L can be found in Figure 3. In equation (11) d_c is the distance between the effective work surface and root of the gear tooth. Its value varies in between 0 to d_h.

2.4 Mesh stiffness of gear with a spalled tooth

Spalling is one of the common surface fatigue failures for gear tooth. It involves the formation of small subsurface cracks which subsequently grow and break out of the contact surface. This phenomenon decreases the contact area of the two meshing gear teeth. Due to this reduction in contact area, stiffness of gear tooth at that point suddenly decreases. In this study we have assumed that the depth of spalled tooth is around 100μm [8] which will show negligible changes in the area and moment of inertia at that section. So the effect of spall depth is neglected in this study.

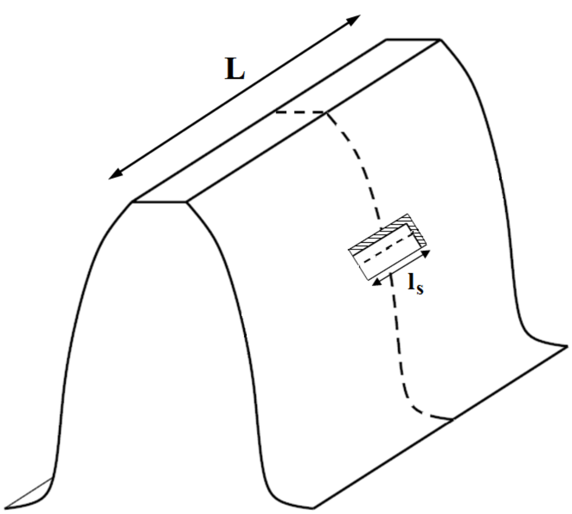

Figure 4. Schematic diagram of spalled tooth

As the change in area and moment of inertia due to spall at that point is not considered, only the length of spall is considered as an effecting parameter responsible for mesh stiffness variation. For a rectangular spall of length 'l_s' as shown in Figure 4 it is assumed that no tooth contact occurs in the spalled area which will directly affect Hertzian contact stiffness. As shown in Figure 4, width of tooth is 'L' and length of rectangular spall is 'l_s' so the effective length (L_s) at that point is

$$L_s = L - l_s \tag{12}$$

Due to this change in length, change in Hertzian stiffness at that point will be

$$K_{h\_spall} = \frac{\pi E L_s}{4(1-\nu^2)} \tag{13}$$

3. Computer simulation of TVMS for gear faults

The main parameters of spur gear pair used in the computer simulation for calculation of TVMS for gear faults are given in Table 1.

Table 1. Main parameters of spur gear pair

Parameter	Value
Young's modulus	$E = 2.068 \times 10^{11}$ Pa
Poisson's ratio	$\nu = 0.3$
Pressure angle	20^0
Width of teeth	L = 0.016 m
Number of teeth on pinion	$N_1 = 19$
Number of teeth on gear	$N_2 = 48$
Base radius of pinion	R_{b1}=0.02834 m
Base radius of gear	R_{b2}=0.0716 m

3.1 TVMS studies of cracked gear tooth

The calculation of mesh stiffness of pinion with a cracked tooth is done as discussed in Section 2.2. Equations (7) - (9) are used to calculate results. In this study we have considered the case when 'q_1' (crack length) is less than the half of base chordal tooth thickness. The crack intersection angle 'ν' is taken as constant between the crack and the central line of the tooth and its value is taken as 45^0. Figure 5 shows the effect of crack on TVMS of gear pair. The effect of change in crack length is also studied in Figure 5 and it can be observed that as the crack length increases, the drop in mesh stiffness value increases drastically. This decrease in TVMS value will increase the vibration of dynamic system.

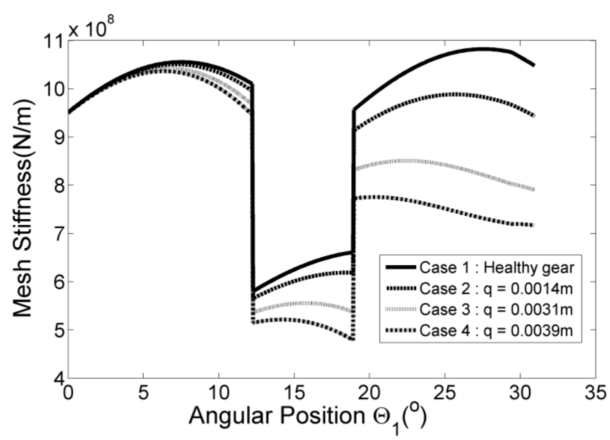

Figure 5. Effect of crack length on TVMS of gear pair

Mesh Stiffness (N/m)

3.2 TVMS studies of chipped gear tooth

The calculation of mesh stiffness of pinion with a chipped tooth is done as discussed in Section 2.3. Equations (10) and (11) are used to calculate results. In this study size of chip is varied i.e. chip height and chip width are varied and their effect on gear mesh stiffness is studied. Figure 6 shows the effect of change in chip height 'z' on TVMS of gear pair. Similarly Figure 7 shows the effect of change in chip width 'b' on TVMS of gear pair. On visual examination of the obtained results we can say that chip formation on pinion gear tooth reduces the mesh stiffness of gear pair and causes increase in vibrations.

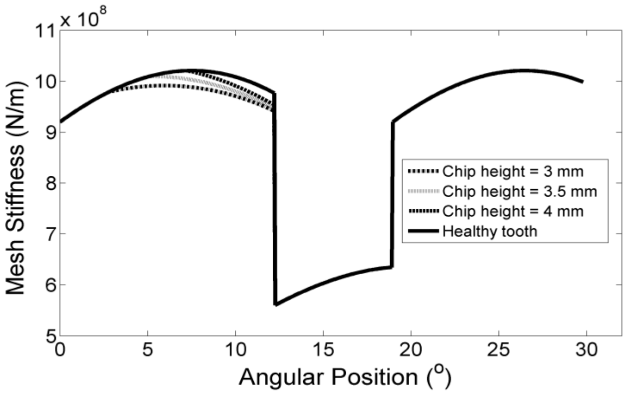

Figure 6. Effect of chip height on TVMS of gear pair

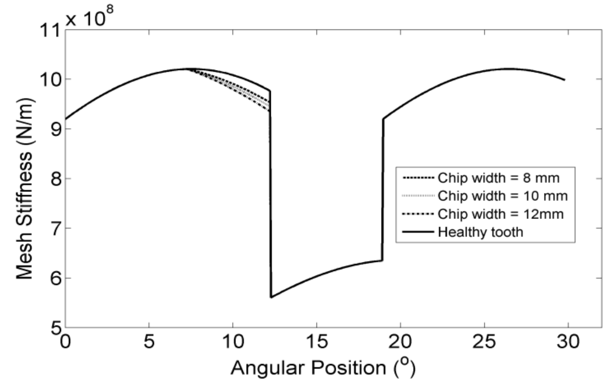

Figure 7. Effect of chip width on TVMS of gear pair

3.3 TVMS studies of spalled gear tooth

The calculation of mesh stiffness of pinion with a spalled gear tooth is done as discussed in Section 2.4. Equations (12) and (13) are used to calculate results. Effect of spall formation is shown in Figure 8 at various locations on gear tooth considering regular spall depth of 0.1 mm, neglecting its effect on the results. We have considered three positions (i) in double point contact (DPC), (ii) in between double point contact and pitch point, (iii) in single point contact (SPC). A reduction in stiffness is observed for all the three positions. These results show that when spall is present at the contact position it will decrease the stiffness value and hence the vibrations of the gear system increase.

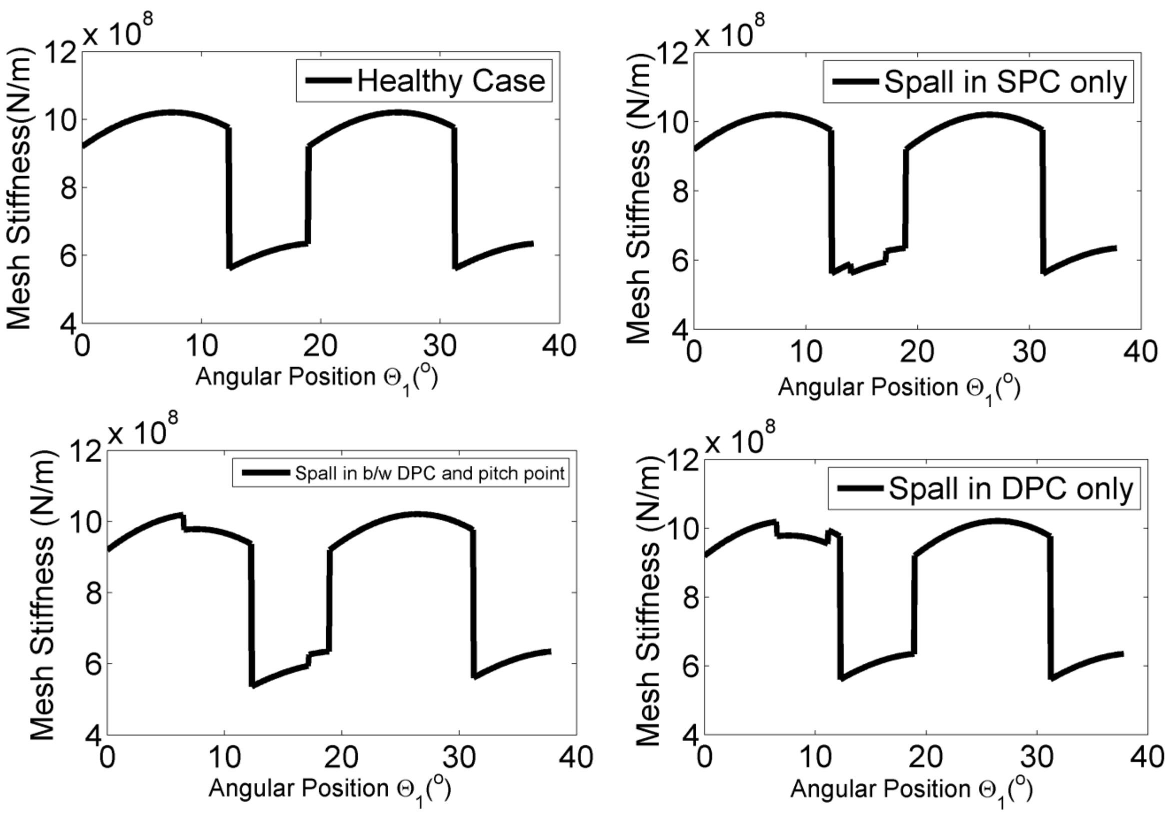

Figure 8. Effect of spall position on TVMS of gear pair

4. Conclusion

In this paper an analytical formulation is adopted to study the effect of variation in mesh stiffness due to the presence of various gear faults. The proposed method allows us to further study the vibration responses due to reduction in mesh stiffness observed due to these faults. It has been found that for defected gears, the values of total effective mesh stiffness have changed as compared to healthy gear. The results show that for cracked tooth, variation in mesh stiffness is observed for complete mesh cycle. Whereas, in case of chipped tooth and spalled tooth, the variation in stiffness is localized. In future work the effect of fillet foundation deflection will also be considered on gear mesh stiffness.

5. References

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Biographical sketch of authors

Ankur Saxena is Ph.D. scholar in the department of mechanical engineering at IIT Indore. His research focus is on condition monitoring of gear systems, vibration analysis of mechanical systems.

Dr. Anand Parey is an associate professor in the department of mechanical engineering at IIT Indore. He holds a doctorate degree from IIT Delhi in Mechanical Engineering. His research focus is on condition monitoring, noise and vibration isolation and in signal processing of mechanical systems.

Dr. Manoj Chouksey is an assistant professor in the department of mechanical engineering at SGSITS Indore. He holds a doctorate degree from IIT Delhi in Mechanical Engineering. His research focus is on modal analysis and modal updating of mechanical systems.